

Chapter 5 – Analytic Trigonometry

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Chapter 5: Analytic Trigonometry

Topic 1: Verifying Trig Identities

Fundamental Identities

These should already be in your memory; otherwise this topic will be a struggle. Strive to commit the following relationships to memory.

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Ratio Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Using Fundamental Identities

By using a mix of the above identities, we will attempt to verify other given identities. To verify an identity, we show that one side of the equation is equal to the other.

General Tips:

- When possible, 'anchor' to cosine and sine.
- If equations, work with each side independently, starting with the more difficult-looking side.
- There may be more than one right way, so your work doesn't have to match mine or a classmates in many cases.
- If you find yourself down an unhelpful path, restart.

Express each of the following as a single term containing one function.

1) $\sin x (\cot x)$

2) $\csc \beta (\tan \beta)$

3) $\sec A \cos^2 A$

4) $\sin \theta \csc \theta \cot \theta$

5) $\sec \theta \cot \theta \sin \theta$

6) $(\cos x)(\tan x)(\csc x)$

7) $2 - 2 \cos^2 \theta$

8) $\cos x(1 + \tan^2 x)$

9) $\cos \theta \sec \theta - \cos^2 \theta$

10) $\sec \theta(1 - \sin^2 \theta)$

11) $\sec \theta \cos \theta - \cos^2 \theta$

12) $\tan \theta (\cot \theta + \tan \theta)$

13) $(1 + \csc \theta)(1 - \csc \theta)$

14) $\frac{\cos \theta}{\sec \theta}$

15) $\frac{\csc \theta}{\cot \theta}$

16) $\frac{\tan \theta}{\sec \theta}$

17) $\frac{\sin^2 \theta - 1}{\cos \theta}$

Prove these identities to be true

1) $\sec \theta \cot \theta = \csc \theta$

2) $\csc \beta \tan \beta = \sec \beta$

3) $\sin \theta \tan \theta + \cos \theta = \sec \theta$

4) $\cos x \cot x + \sin x = \csc x$

Factoring can be helpful on the more complicated looking side:

5) $\cos \theta - \cos \theta \sin^2 \theta = \cos^3 \theta$

6) $\sin x - \sin x \cos^2 x = \sin^3 x$

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Chapter 5: Analytic Trigonometry
Topic 1: Homework

Verify the Trig Identities

1. $\sin x \sec x = \tan x$

2. $\cos x \csc x = \cot x$

3. $\tan(-x)\cos x = -\sin x$

4. $\cot(-x)\sin x = -\cos x$

5. $\tan x \csc x \cos x = 1$

6. $\cot x \sec x \sin x = 1$

7. $\sec x - \sec x \sin^2 x = \cos x$

8. $\csc x - \csc x \cos^2 x = \sin x$

9. $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$

10. $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1$

11. $\csc \theta - \sin \theta = \cot \theta \cos \theta$

12. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

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Topic 2: Double Angle Formulas**Double Angle Formulas:**

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

1. If $\sin \theta = \frac{5}{13}$ and θ terminates in QII, find the exact value of:

a. $\cos 2\theta$

b. $\sin 2\theta$

c. $\tan 2\theta$

2. If $\sin \theta = \frac{4}{5}$ and θ terminate in QII, find the exact value of:

a. $\sin 2\theta$

b. $\cos 2\theta$

c. $\tan 2\theta$

3. Find the exact value of $\frac{2 \tan 15^\circ}{1 - \tan^2 15^\circ}$

4. Find the exact value of $\cos^2 15^\circ - \sin^2 15^\circ$

Verify the Identities:

1. $\cos 3\theta = \cos(2\theta + \theta)$

2. $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

3. $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

4. $(\sin(\theta) + \cos(\theta))^2 = 1 + \sin(2\theta)$

$$5. \sin^2 x + \cos 2x = \cos^2 x$$

$$6. \cot x = \frac{\sin 2x}{1 - \cos 2x}$$

$$7. \sin 2t - \tan t = \tan t \cos 2t$$

$$8. \sin 4t = 4 \sin t \cos^3 t - 4 \sin^3 t \cos t$$

Chapter 5: Analytic Trigonometry
Topic 2: Homework

In Exercises 15–22, write each expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

15. $2 \sin 15^\circ \cos 15^\circ$

16. $2 \sin 22.5^\circ \cos 22.5^\circ$

17. $\cos^2 75^\circ - \sin^2 75^\circ$

18. $\cos^2 105^\circ - \sin^2 105^\circ$

19. $2 \cos^2 \frac{\pi}{8} - 1$

20. $1 - 2 \sin^2 \frac{\pi}{12}$

21.
$$\frac{2 \tan \frac{\pi}{12}}{1 - \tan^2 \frac{\pi}{12}}$$

22.
$$\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$$

Chapter 5: Analytic Trigonometry
Topic 3: Trigonometric Equations (DAY 1)

Basic Solving Instructions:

Solve for θ : $\cos \theta = -\frac{\sqrt{2}}{2}$

1) $RA = 45^\circ$

2) Negative

| QII | QIII |
|-------------------------------------|----------------------|
| $\theta = 180 - RA$ | $\theta = 180 + RA$ |
| $\theta = 180 - 45$ | $\theta = 180 + 45$ |
| $\theta = 135^\circ$ | $\theta = 225^\circ$ |
| $\theta = \{135^\circ, 225^\circ\}$ | |

Steps:

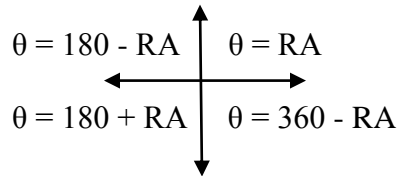
1) Determine the R.A. where the \cos of $\theta = \frac{\sqrt{2}}{2}$

(IGNORE THE SIGN)

2) write the sign (look at the isolated equation)

3) write the 2 quadrants where \cos is negative

4) write the rules for each Quadrant



5) Substitute and solve for θ

6) Write solution set and check in your calculator

Solve each equation for all values of θ in the interval $0^\circ < \theta \leq 360^\circ$

1) $\tan \theta = 1$

2) $\sin \theta = -\frac{1}{2}$

3) $\cos \theta = -\frac{1}{2}$

4) $\sin \theta = \frac{\sqrt{2}}{2}$

5) $\tan \theta = -\sqrt{3}$

6) $\cos \theta = \frac{\sqrt{3}}{2}$

Solve each equation for all values of θ in the interval $0 \leq \theta \leq 2\pi$

$$7) \cos \theta = -\frac{\sqrt{2}}{2}$$

$$8) \sin \theta = \frac{\sqrt{3}}{2}$$

$$9) \tan \theta = \frac{\sqrt{3}}{3}$$

$$10) \cot \theta = -\sqrt{3}$$

$$11) \sec \theta = -\sqrt{2}$$

$$12) \csc \theta = \frac{2\sqrt{3}}{3}$$

Solve each equation for all values of θ in the interval $[0^\circ, 360^\circ)$

For these questions you will first have to isolate the function. Also, the Reference Angle is not always going to be a special angle (from the chart) to solve these you will have to use the arc function (ex: \tan^{-1})

Ex: $5\cos \theta + 6 = 3$

$$5\cos \theta = \underline{-3}$$

$$\cos \theta = -\frac{3}{5}$$

$$\text{RA} = \cos^{-1}\left(\frac{3}{5}\right)$$

$$\text{RA} \approx 53.13^\circ$$

$$\text{RA} \approx 53^\circ \quad (\text{always round to the nearest degree, unless otherwise noted})$$

Now you continue with steps 2 – 6.

1) $2\sin \theta + \sqrt{3} = 0$

2) $4\cos \theta - 2\sqrt{2} = 0$

3) $3\tan \theta + 5 = 2$

4) $7\sin \theta + 12 = 8$

5) $6\csc \theta - 15 = 2$

6) $3(2\sec \theta + 4) = 19$

Chapter 5: Analytic Trigonometry
Topic 3: Homework

Find all solutions to the equations.

11. $\sin x = \frac{\sqrt{3}}{2}$

13. $\tan x = 1$

15. $\cos x = -\frac{1}{2}$

17. $\tan x = 0$

19. $2 \cos x + \sqrt{3} = 0$

12. $\cos x = \frac{\sqrt{3}}{2}$

14. $\tan x = \sqrt{3}$

16. $\sin x = -\frac{\sqrt{2}}{2}$

18. $\sin x = 0$

20. $2 \sin x + \sqrt{3} = 0$

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Topic 3: Trigonometric Equations (DAY 2)

Solve each equation below for all values of the angle in the interval $0 \leq \theta < 360^\circ$

Steps:

First Factor the equation (you may use the Quadratic formula). Remember when you are factoring, you are NOT solving for θ , you are solving for the function of θ . Such as $\sin \theta$ or $\cos \theta$. You should have two different linear equations.

Now follow steps 1- 6 for solving linear trig equations

Ex: $\cos^2\theta + 2\cos\theta = 3$

$$\frac{\cos^2\theta + 2\cos\theta - 3 = 0}{(\cos\theta + 3) | (\cos\theta - 1) = 0}$$

| | |
|----------------------|----------------------|
| $\cos\theta + 3 = 0$ | $\cos\theta - 1 = 0$ |
| $\cos\theta = -3$ | $\cos\theta = 1$ |

| | |
|--|---------------------------------|
| X out of range $-1 \leq \cos \theta \leq 1$ | Quadrantal $\theta = 0, 360$ |
|--|---------------------------------|

$\theta = \{0\}$

$$2\sin^2\theta - 5\sin\theta + 2 = 0$$

$$2\sin^2\theta - 4\sin\theta - 1\sin\theta + 2 = 0$$

$$2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$$

$$(2\sin\theta - 1) | (\sin\theta - 2) = 0$$

| | |
|----------------------------|----------------------|
| $2\sin\theta - 1 = 0$ | $\sin\theta - 2 = 0$ |
| $\sin\theta = \frac{1}{2}$ | $\sin\theta = 2$ |

| | |
|-------------|------------------------------|
| 1) RA = 30 | X out of range |
| 2) positive | $-1 \leq \sin \theta \leq 1$ |

| | |
|----------------------|----------------------------|
| 3) I | II |
| $\theta = \text{RA}$ | $\theta = 180 - \text{RA}$ |
| $\theta = 30$ | $\theta = 180 - 30$ |
| | $\theta = 150$ |

$\theta = \{30^\circ, 150^\circ\}$

1. $\sin^2\theta + 7\sin\theta - 8 = 0$

2. $\cos^2\theta - 7\cos\theta + 12 = 0$

$$3. \tan^2\theta - 6\tan\theta = 16$$

$$4. \cos^2\theta + 25\cos\theta + 24 = 0$$

$$5. \sin^2\theta + \sin\theta \cos\theta = 0$$

$$6. \cos\theta = \tan\theta\cos\theta$$

$$7. 6\sin^2\theta + 7\sin\theta = 3$$

$$8. 4\cos^2\theta = 21\cos\theta - 5$$

$$9. 5\tan^2\theta - 33\tan\theta - 14 = 0$$

$$10. \tan^2\theta - 6\tan\theta - 11 = 0$$

Chapter 5: Analytic Trigonometry
Topic 3: Homework

Solve the trig equations.

39. $2 \sin^2 x - \sin x - 1 = 0$

40. $2 \sin^2 x + \sin x - 1 = 0$

41. $2 \cos^2 x + 3 \cos x + 1 = 0$

42. $\cos^2 x + 2 \cos x - 3 = 0$

43. $2 \sin^2 x = \sin x + 3$

44. $2 \sin^2 x = 4 \sin x + 6$

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Topic 3: Trigonometric Equations (DAY 3)

We can use Pythagorean, and Double Angle Identities to solve the equations.

Solve each equation for θ such that $0 < \theta \leq 360^\circ$.

1. $\cos 2\theta = -1$

2. $\cos 2\theta - \cos \theta = 0$

3. $\sin 2\theta \sin \theta = \cos \theta$

4. $\cos 2\theta = -2\cos^2 \theta$

5. $\cot x = \tan 2\theta$

6. $\sin 2\theta + \sin \theta = 0$

$$7. 2\tan\theta = \tan 2\theta$$

$$8. \sin 2\theta = \tan\theta$$

$$9. \cos 2\theta - 3\sin\theta = 2$$

$$10. \cos^2\theta + \sin\theta = 1$$

$$11. \sec^2\theta - \tan\theta - 1 = 0$$

$$12. 1 + \cot^2\theta = 2\csc\theta - 1$$

$$13. 2\cos\theta = \sec\theta$$

$$14. \cot\theta = 3\tan\theta$$

$$15. 2\cos^2\theta - \sin\theta - 1 = 0$$

$$16. 4\sin^2\theta + 4\cos\theta = 5$$

Chapter 5: Analytic Trigonometry
Topic 3: Homework

Solve the trig equations.

63. $2 \cos^2 x + \sin x - 1 = 0$ **64.** $2 \cos^2 x - \sin x - 1 = 0$

65. $\sin^2 x - 2 \cos x - 2 = 0$

66. $4 \sin^2 x + 4 \cos x - 5 = 0$

67. $4 \cos^2 x = 5 - 4 \sin x$ **68.** $3 \cos^2 x = \sin^2 x$

69. $\sin 2x = \cos x$ **70.** $\sin 2x = \sin x$

71. $\cos 2x = \cos x$ **72.** $\cos 2x = \sin x$

Chapter 5: Analytic Trigonometry
Topic 3: Trigonometric Equations (DAY4)

Practice.

Solve each equation for θ such that $0 < \theta \leq 360^\circ$.

1. $\tan \theta \sin^2 \theta = 3 \tan \theta$

2. $\cos 2\theta + 3 \sin \theta - 2 = 0$

3. $\cos \theta = 2 \sin \theta \cos \theta$

4. $\sin \theta - \cos \theta = 1$

5. $3 \sin \theta - 2 = 5 \sin \theta - 1$

6. $\cos \theta - \sin \theta = -1$

$$7. \cos 2\theta + \sin \theta = 0$$

$$8. \sin \theta \cos \theta = -\frac{1}{2}$$

$$9. 2 \cos^2 \theta + \cos \theta - 1 = 0$$

$$10. 5 \tan^2 \theta + 3 \tan \theta = 2$$

$$11. 4 \sin \theta = 2 \sin \theta - 1$$

$$12. 2 \cos \theta + 4(\cos \theta - 7) = \cos \theta - 30$$

$$13. \sin^2 \theta - 2\sin \theta = 3$$

$$14. \tan^2 \theta = -\tan \theta$$

$$15. 3(\tan \theta + 2) = 2\tan \theta + 3$$

$$16. 3\sin^2 \theta + \sin \theta + 5 = 4(1 - \sin \theta)$$

Chapter 5: Analytic Trigonometry
Topic 3: Homework

Solve the trig equations.

73. $\cos 2x + 5 \cos x + 3 = 0$ 74. $\cos 2x + \cos x + 1 = 0$

75. $\sin x \cos x = \frac{\sqrt{2}}{4}$ 76. $\sin x \cos x = \frac{\sqrt{3}}{4}$

77. $\sin x + \cos x = 1$ 78. $\sin x + \cos x = -1$

79. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$

80. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

81. $\sin 2x \cos x + \cos 2x \sin x = \frac{\sqrt{2}}{2}$

82. $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$

83. $\tan x + \sec x = 1$ 84. $\tan x - \sec x = 1$